

Exercise 2

Sketch the set of points determined by the condition

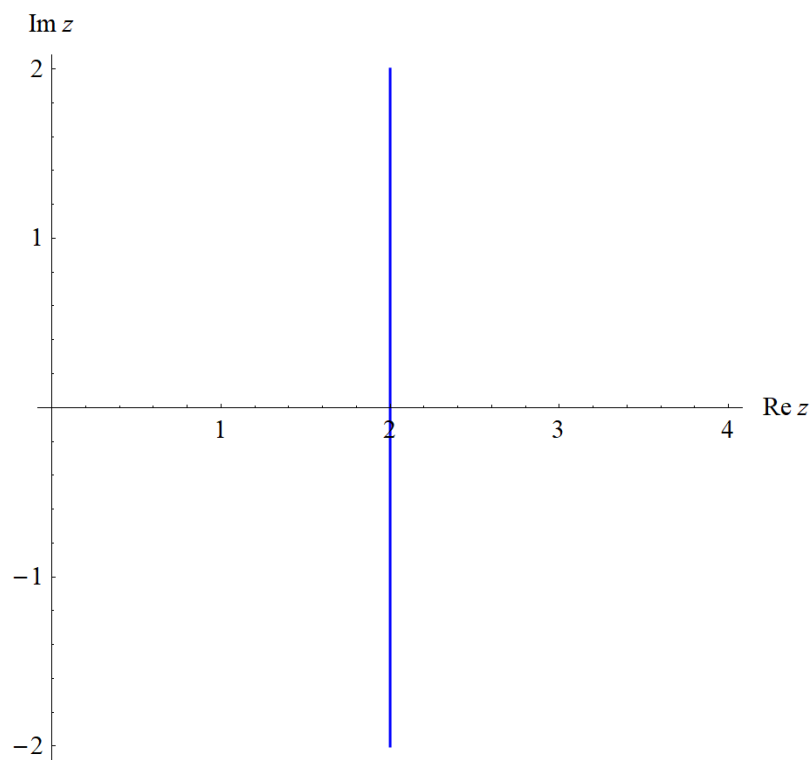
$$(a) \operatorname{Re}(\bar{z} - i) = 2; \quad (b) |2\bar{z} + i| = 4.$$

Solution

Part (a)

$$\begin{aligned}\operatorname{Re}(\bar{z} - i) &= 2 \\ \frac{\bar{z} - i + \overline{\bar{z} - i}}{2} &= 2 \\ \frac{\bar{z} - i + \bar{\bar{z}} - \bar{i}}{2} &= 2 \\ \frac{\bar{z} - i + z + i}{2} &= 2 \\ \frac{z + \bar{z}}{2} &= 2 \\ \operatorname{Re} z &= 2\end{aligned}$$

The graph is therefore a straight line.



Part (b)

$$|2\bar{z} + i| = 4$$

$$|\overline{2z - i}| = 4$$

$$|2z - i| = 4$$

$$|2| \left| z - \frac{i}{2} \right| = 4$$

$$2 \left| z - \frac{i}{2} \right| = 4$$

$$\left| z - \frac{i}{2} \right| = 2$$

The graph is therefore a circle centered at $(0, 1/2)$ with radius $\sqrt{2}$.

